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## C(OpenCV



Useful MCMC packages: OpenBUGS, RJAGS, RStan
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## Gy Jython




## ilastik

the interactive learning and segmentation toolkit



## BE/Bi 103

Data Analysis in the Biological Sciences
Fall term, 2015

## The scientific method



## Statistical inference requires a probability theory

$M_{i}$ : model $i$
$\mathrm{a}_{i}$ : the set of parameters associated with model $i$
$D$ : the measured data
I: all other knowledge

Bayes's theorem for parameter estimation:

$$
\text { posterior }=P\left(\mathrm{a}_{i} \mid D, M_{i}, I\right)=\frac{P\left(D \mid \mathrm{a}_{i}, M_{i}, I\right) P\left(\mathrm{a}_{i} \mid M_{i}, I\right)}{P\left(D \mid M_{i}, I\right)}=\frac{\text { likelihood } \cdot \text { prior }}{\text { evidence }}
$$

Normalization of posterior (marginalization):

$$
P\left(D \mid M_{i}, I\right)=\int \mathrm{d} \mathbf{a} P\left(D \mid \mathbf{a}_{i}, M_{i}, I\right) P\left(\mathbf{a}_{i} \mid M_{i}, I\right)
$$

Bayes's theorem for model selection:

$$
P\left(M_{i} \mid D, I\right)=\frac{P\left(D \mid M_{i}, I\right) P\left(M_{i} \mid I\right)}{P(D \mid I)}
$$

# Model type 1: Cartoons (informal) 

Model a:


Model b:


Model type 2: Mathematized cartoons (formal)

Model a:


$$
\begin{aligned}
& l \neq l(d) \\
& l=\theta
\end{aligned}
$$

Model b:


$$
l(d ; \gamma, \theta)=\frac{\gamma d}{\left(1+(\gamma d / \theta)^{3}\right)^{\frac{1}{3}}}
$$

# Model type 2: Mathematized cartoons (formal) 



# Model type 3: Model type 2 + data description 

Model a:

$l_{i}=\theta+e_{i}$
$e_{i}$ Gaussian distributed

Model b:


$$
l_{i}=\frac{\gamma d_{i}}{\left(1+\left(\gamma d_{i} / \theta\right)^{3}\right)^{\frac{1}{3}}}+e_{i}
$$

$e_{i}$ Gaussian distributed

## Models (definition 3!)

$M_{i}$ and $I$ encode the functional form of the likelihood $P\left(D \mid \mathrm{a}_{i}, M_{i}, I\right)$ and prior $P\left(\mathbf{a}_{i} \mid M_{i}, I\right)$.

Prior $P\left(\mathrm{a}_{i} \mid M_{i}, I\right)$ : Often chosen to be uninformative, e.g., uniform or Jeffreys.
Likelihood $P\left(D \mid \mathrm{a}_{i}, M_{i}, I\right)$ : Depends on model, often independent Gaussians.

Given the model and all our previous knowledge, the posterior is completely determined. All of the "work" of inference is computing it!

## Computing the posterior: analytical results

Multiple measurements of parameter $\mu$ with unknown variance $\sigma^{2}$.

$$
\begin{aligned}
P\left(\left\{x_{i}\right\} \mid \mu, \sigma, I\right) & =\prod_{i} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \mathrm{e}^{-\left(x_{i}-\mu\right)^{2} / 2 \sigma^{2}} \\
P(\mu, \sigma \mid I) & \propto \sigma^{-1}
\end{aligned}
$$

$$
\begin{aligned}
& \text { most probable } \mu=\bar{x} \equiv \frac{1}{n} \sum_{i} x_{i} \\
& \text { most probable } \sigma^{2}=r^{2} \equiv \frac{1}{n} \sum_{i}\left(x_{i}-\bar{x}\right)^{2} \\
& P\left(\mu \mid\left\{x_{i}\right\}, I\right) \approx \frac{\Gamma\left(\frac{n}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{n-1}{2}\right)} \frac{1}{r}\left(1+\frac{(\bar{x}-\mu)^{2}}{r^{2}}\right)^{-\frac{n}{2}} \quad \text { (Student-t) } \\
& \mu \approx \bar{x} \pm r / \sqrt{n}
\end{aligned}
$$

## Computing the posterior: analytical results




## Computing the posterior: approximate summary

1. Find most probable parameters $\mathbf{a}^{*}$.
2. Approximate $P(\mathbf{a} \mid D, I)$ as Gaussian by doing a Taylor expansion of $\ln P(\mathbf{a} \mid D, I)$ about $a^{*}$.
3. The covariance matrix is given by the negative inverse of the Hessian of In $P(\mathbf{a} \mid D, I)$.

Obvious assumption: posterior is approximately Gaussian.

## Computing the posterior: approximate summary



## Computing the posterior: MCMC

1. Define the (log) posterior distribution.
2. Efficiently sample the posterior with an ergodic, positively recurrent Markov chain.
3. Posterior is trivially marginalized by considering specific parameters.
4. Bin samples to get histograms describing posterior.

## Computing the posterior: MCMC




## Foray into frequentism




## DID THE SUN JUST EXPLODE? <br> ( TTS NGHT, SO WERE NOT SURE.)



FREQUENTIST STATISTCIAN:


BAYESIAN STATSTICAN:


## Image segmentation



## Image segmentation




## Colocalization







